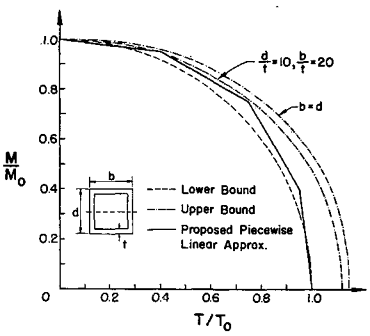

Fig. 2 Inelastic interaction curves for thin-walled box sections

BC, *CD*, and *DE* passing closely through the theoretically computed points. The flow rule and the energy-dissipation rate associated with this approximation can be determined easily by considering the normality condition of the strain-rate vector.

The new interaction curve is also valid for thin-walled box beams. Figure 2 shows a comparison of this curve with the upper- and lower-bound solutions obtained by Gaydon and Nuttall.[6] The proposed interaction curve closely approximates the lower-bound solution and is everywhere within the upper bounds.

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Scaling of Jet Flameholders

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The problem of scaling when the flame stabilizer is a gasjet is considered. This study requires that the value of certain factors such as inlet temperatures and equivalence ratios be equal in model and prototype if scaling is to be considered. As is usually the case, some dimensionless parameters must be allowed to vary during scaling. The requirement that the mainstream-jet interaction be retained during scaling is used in selecting the mainstream Mach number rather than Reynolds number as a scaling parameter. The variation of these two factors during scaling is also considered in reaching this decision. The other scaling parameters are the pressure ratio across the jet and the ratio of residence time to reaction time in the combustor.

Nomenclature

A = area
c = constant
F = stoichiometric fuel-air ratio
L = characteristic dimension
$\dot{m}$ = mass rate of flow
M = Mach number
n = effective overall order of reaction
P = static pressure
q = dynamic pressure
Re = Reynolds number
T = temperature
V = velocity
w = rate of reaction per unit volume
η = combustion efficiency
ρ = density
τ = dimensionless time ratio
φ = equivalence ratio

Subscripts

c = critical zone
cj = into critical zone from jet
cs = into critical zone from mainstream
j = jet
m = model
p = prototype
s = mainstream

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A STREAM of air or air-fuel mixture injected upstream in a flow of combustible gas has been found[1] to have the ability to stabilize a flame in this flow. Such a flame-stabilizing mechanism is commonly called a jet flameholder to contrast it to the more common bluff-body type.

Studies of the jet flameholder have been largely experimental because of the complexity of the flameholding mechanism and the large number of variables involved.

This note is an effort to assist in the use of the jet flameholder through a study of similitude parameters to allow scaling from a successful test model to a full scale prototype. A knowledge of scaling is desirable, since both turbojet afterburners and ramjet combustion chambers, where flameholders are used, can be large in size.

The combustion efficiency will be investigated as a function of the various dimensionless parameters. Other combustion chamber dimensionless dependent variables, such as $\Delta T/T_s$ or $\Delta P/P_s$, also are covered by the discussion, as they are functions of the same dimensionless parameters as is η.

Under the following conditions (which apply throughout this note): 1) fixed fuel, 2) fixed equivalence ratios in mainstream and jet, 3) fixed mainstream and jet temperatures, and 4) fixed fuel temperatures, the combustion efficiency for a group of geometrically similar arrangements can be written as

$$\eta = f_1(M_s, M_j, Re_s, Re_j, \tau) \tag{1}$$

Way[2] has suggested a method of handling the dimensionless time ratio τ. He suggests writing the rate of reaction in the form of an effective overall nth order reaction.

$$w = c\rho^n \tag{2}$$

Equation (2) is valid for constant inlet temperatures and fuel composition providing that the usual complicated chain reaction kinetics can be described accurately by a single nth order reaction.

The dimensionless time ratio, which is residence time divided by reaction time, is written as

$$\tau = \left(\frac{L/V}{\rho/w}\right)_s = \left(\frac{Lw}{\rho V}\right)_s = \left(\frac{cL\rho^{n-1}}{V}\right)_s \tag{3}$$

In the case of a jet flameholder, the factors used in Eq. (3) are those of the mainstream, since the mainstream conditions are more important provided that the interaction between jet and mainstream is fixed. There are thus three similitude parameters for the mainstream in Eq. (1). As pointed out in Ref. 2, setting the value of these three parameters equal in

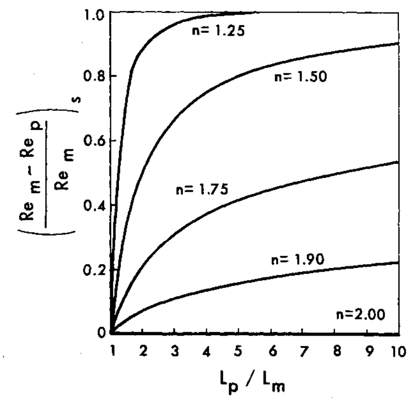

Fig. 1 Variation of Reynolds number between model and prototype with Mach number and time ratio constant

model and prototype excludes scaling except for the case of $n = 2$. Woodward[3] has shown that τ is a very important similitude parameter, possibly the most important of the three from the standpoint of combustion efficiency. Considering these facts, similitude is best achieved in the following manner:

1) Re_s is discarded, whereas M_s and τ are taken as the mainstream similitude parameters. For low M_s, Re_s normally would be retained, but here the mainstream-jet interaction is of great importance. This requires that M_s be retained for all conditions.

2) Re_j is replaced by q_s/q_j. The latter factor is important in the interaction between jet and mainstream.

Equation (1) is now written as

$$\eta = f_2(M_s, M_j, q_s/q_j, \tau) \tag{4}$$

Physically, two of the factors in Eq. (4) can be combined, since they are dependent. The jet Mach number (M_j) is kept the same in model and prototype by using the same ratio of jet manifold to mainstream pressure (P_j/P_s) in both. This, however, along with the temperature and Mach number restrictions, means that ρ_j/ρ_s is the same in model and prototype. Equation (4) can therefore be written as

$$\eta = f_3(M_s, P_j/P_s, \tau) \tag{5}$$

It will be shown that the use of the factors in Eq. (5) as similitude parameters will provide the best scaling. An important reason for this is that many ratios that are important in the interaction between the jet and mainstream are equal in model and prototype under these conditions.

Another reason for selecting M_s and τ as scaling parameters rather than Re_s and τ is illustrated by Figs. 1 and 2.

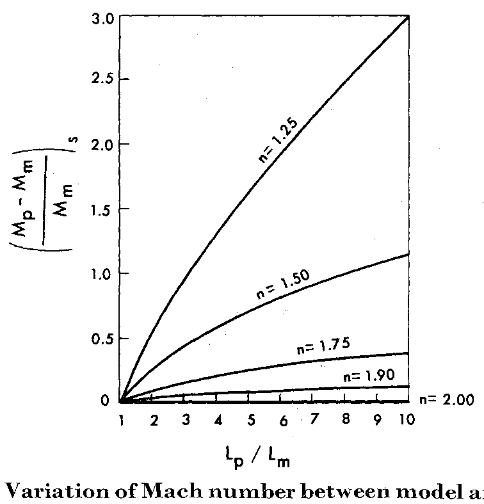

Fig. 2 Variation of Mach number between model and prototype with Reynolds number and time ratio constant

With M_s and τ constant in model and prototype, the Reynolds number variation between the two arrangements can be written as

$$\left(\frac{Re_m - Re_p}{Re_m}\right)_s = 1 - \left(\frac{L_p}{L_m}\right)^{(2-n)/(1-n)} \tag{6}$$

Equation (6) is used to plot Fig. 1.

For the condition of Re_s and τ the same in model and prototype, the variation in Mach number can be written as

$$\left(\frac{M_p - M_m}{M_m}\right)_s = \left(\frac{L_p}{L_m}\right)^{(2-n)/n} - 1 \tag{7}$$

Figure 2 is plotted from Eq. (7).

A comparison between these figures reveals that the variation in Re_s for the same scaling factor (L_p/L_m) and effective overall reaction order (n) is much smaller than that of M_s. Thus, similitude should be better achieved from this point of view by allowing Re_s rather than M_s to vary.

Several other facts are worth noting from Figs. 1 and 2. The importance of large-scale testing is apparent. However, since small-scale testing is advisable economically, some compromise must be made, and for the similitude parameters of Eq. (4), Fig. 1 should aid in this decision. It should be noticed that the fuel being used is important, as it (as well as all other factors that affect the reaction kinetics) exerts a strong influence on the value of n.

It is in order to examine the similitude benefits of Eq. (4) [which is essentially the same as Eq. (5) as previously discussed]. Recalling the conditions listed previous to Eq. (1), it is apparent that Eq. (4) requires that the following exist between model and prototype:

$$V_s = \text{const} \tag{8}$$

$$V_j = \text{const} \tag{9}$$

$$\rho_s/\rho_j = \text{const} \tag{10}$$

$$(L\rho^{n-1})_s = \text{const} \tag{11}$$

It can be shown easily with the aid of Eqs. (8–11) that the factors $\dot{m}_s/\dot{m}_j$, V_s/V_j, and $(\dot{m}/A)_s/(\dot{m}/A)_j$ are the same in model and prototype. Thus the set of parameters in Eq. (4) should maintain accurately the same interaction between the jet and the mainstream, since the factors here mentioned include those of primary importance in this interaction.

Schaffer and Cambel[4] have suggested that a critical zone exists near the stagnation region caused by an opposed jet. They have presented a straightforward equation for the equivalence ratio in this critical zone which in the present notation can be written as

$$\varphi_c = \frac{(\dot{m}_{cs}/\dot{m}_{cj})[\varphi_s/(1 + F\varphi_s)] + [\varphi_j/(1 + F\varphi_j)]}{(\dot{m}_{cs}/\dot{m}_{cj})[1/(1 + F\varphi_s)] + [1/(1 + F\varphi_j)]} \tag{12}$$

This equation is valid regardless of the location of the critical zone. It assumes only that any recirculated gas entering the critical zone is inert. It is desirable that φ_c be the same in model and prototype. It has been required previously that φ_j, φ_s, and F be held constant during scaling. Thus φ_c will be fixed if $\dot{m}_{cs}/\dot{m}_{cj}$ is the same in model and prototype. It is very difficult to investigate this ratio, because the critical zone itself is not well defined. It would seem, however, that with the requirements of geometric similarity and Eqs. (8–11), which require constancy of the factors q_s/q_j, $\dot{m}_s/\dot{m}_j$, V_s/V_j, and $(\dot{m}/A)_s/(\dot{m}/A)_j$, the ratio $\dot{m}_{cs}/\dot{m}_{cj}$ would be very nearly the same in model and prototype.

Thus, although other groups of similitude parameters were investigated, it is felt that, in the light of this study, geometrically similar jet flameholders are most accurately scaled by the use of the factors in the right-hand side of Eq. (4). Physically, this requires constancy of the factors in the right-hand side of Eq. (5).

A recent investigation by Hottel et al.[5] indicates that the exterior heat loss also can be important in scaling of combustors. If this loss from the prototype is large, the additional similitude parameter discussed in Ref. 5 should be considered. Further problems are encountered if liquid fuel is used, but scaling of liquid fuel systems will not be considered here.

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Lagrangian and Hamiltonian Rocket Mechanics

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The equation of motion of a point-mass rocket in Lagrangian form valid in any coordinate system is derived. The Lagrangian form of the rocket equation can be used conveniently in the calculation of rocket motion involving complicated constraints for which the use of generalized dynamical coordinates is advantageous. As an example, a problem of rocket motion with moving constraints is discussed. The generalized Hamiltonian form of the rocket equation then is derived, and the same problem is worked out in Hamiltonian form.

Lagrangian

CONSIDERING the rocket as a point of variable mass moving with velocity $\dot{x}_i$ along any trajectory, the differential equations of rocket motion are

$$m\ddot{x}_i = \dot{m}c_i + F_i \qquad i = 1, 2, 3 \quad (1)$$

where

t = time; dot above a letter specifies the differentiation with respect to time
$m(t)$ = mass of the rocket at the instant t
$\dot{m}$ = dm/dt = rate of fuel burning
$x_i(t)$ = rectangular Cartesian coordinates of the rocket in fixed coordinate system at the instant t
c_i = velocity of expelled mass relative to the rocket
F_i = any applied force (excluding thrust force $\dot{m}c_i$); usually F_i is the sum of aerodynamic and gravity forces

Adding the term $\dot{m}\dot{x}_i$ to both sides of (1), one obtains

$$(d/dt)(m\dot{x}_i) = \dot{m}u_i + F_i \quad (2)$$

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where $u_i = \dot{x}_i + c_i$ is the absolute velocity of the expelled mass in fixed coordinate system.

The generalized coordinates q_α are introduced by the set of equations

$$x_i = x_i(q_\alpha, t) \qquad i = 1, 2, 3 \qquad \alpha = 1, \ldots, n \quad (3)$$

where n is equal to 1, 2, or 3, depending upon the degrees of freedom of the rocket. If $n = 1$, the rocket is constrained to move along a curve, for $n = 2$ along the surface, and for $n = 3$ the rocket can move freely in space. Since, according to (3), x_i is not only a function of q_α but also of time t, the constraints can move in space.

The kinetic energy of the rocket at any instant t is given by

$$T = \tfrac{1}{2}m(t)\dot{x}_1\dot{x}_i \quad (4)$$

(Summation on repeated indices is implied.) In what follows, only indices i, j, α, and β will be used, whereby it is agreed that i and j will have always the values $i = 1, 2, 3$ and subscripts α and β have always the values $\alpha, \beta = 1, \ldots, n$, where n is the number of degrees of freedom of the rocket.

Dotting Eq. (2) by $\partial x_i/\partial q_\alpha$, one obtains

$$\frac{d}{dt}(m\dot{x}_i)\frac{\partial x_i}{\partial q_\alpha} = \dot{m}u_i\frac{\partial x_i}{\partial q_\alpha} + F_i\frac{\partial x_i}{\partial q_\alpha} \quad (5)$$

It can be verified readily that the left member of (5) can be expressed in the form

$$\frac{d}{dt}(m\dot{x}_i)\frac{\partial x_i}{\partial q_\alpha} = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_\alpha}\right) - \frac{\partial T}{\partial q_\alpha} \quad (6)$$

Hence, Eq. (5) can be written in the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_\alpha}\right) - \frac{\partial T}{\partial q_\alpha} = \dot{m}u_i\frac{\partial x_i}{\partial q_\alpha} + F_i\frac{\partial x_i}{\partial q_\alpha} \quad (7)$$

Equation (7) is the Lagrangian form of the rocket equation in generalized coordinates. The form (7) of the rocket equation can be used conveniently in any problem of rocket motion with moving constraints. The following example is chosen to demonstrate the solution of Eq. (7). A straight line describes a right circular cone above a vertical (x_3 axis) with a uniform angular speed ω. A small rocket of variable mass $m(t)$ moves along this rotating straight line without a friction. By integrating twice Eq. (7) the position of the rocket at any time t will be found.

Let θ be the angle that the straight line makes with the vertical. If x_i are rectangular coordinates of the rocket, then, according to Fig. 1,

$$\begin{aligned} x_1 &= r\sin\theta\cos\omega t \\ x_2 &= r\sin\theta\sin\omega t \\ x_3 &= r\cos\theta \end{aligned} \quad (8)$$

where r is the distance from the apex 0 to the rocket. Since

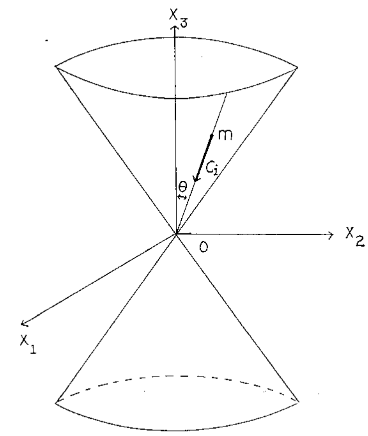

Fig. 1 Motion of rocket along a rotating straight line